

THERMAL THEORY OF HYDRODYNAMIC LUBRICATION IN  
HYDRAULIC EXTRUSION IN THE PRESENCE OF  
HIGH-INTENSITY LONGITUDINAL ULTRASONIC VIBRATIONS

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Heating effects are considered in the hydraulic extrusion of a metal in the presence of longitudinal vibrations.

Ultrasound has a marked effect on the thickness of the lubricant layer in the deformation area during metal extrusion, and it also influences the working pressure in the liquid [1, 2]. The theory of this process has been built up on the assumption that the lubricant film is isothermal, whereas in fact the heat supplied by the ultrasound reduces the viscosity considerably.

The analysis is simplified by assuming that Newtonian flow occurs and that the pressure gradient at the deformation focus is negligible and is zero at the entrance and exit of the plastic-strain zone; also, it is assumed that the film thickness varies linearly in the strain region, while the plastic-strain energy and the part of the ultrasonic energy that enters the strain focus are completely converted to heat, with the plastic-strain heat uniformly distributed over the cross section, while axial heat transfer is negligible [3-5].

The major laws of this form of extrusion may be examined by dividing the system into two zones: 1) the entrance zone and 2) the plastic-strain zone (Fig. 1).

In the second zone, the pressure in the liquid between the metal and the die rises from the source level to the value corresponding to the yield point; the conditions in this zone may be taken as isothermal [6].

The following is the thickness of the lubricant layer in any part of the entrance zone in the (x0y) coordinate system:

$$h = (x - x_0) \operatorname{tg} \alpha. \quad (1)$$

The thickness of the lubricant layer under these circumstances may be determined to a first approximation from the differential equation

$$\frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) = -6\eta \frac{dh}{dx} (V_1 + \bar{V}_t), \quad (2)$$

where  $V_w$  is the mean vibrational speed of the working parts in a half-cycle in which the direction of the longitudinal vibration is that of the general motion of the metal.

Differentiation of the left side in (2) and the substitution  $\eta = \eta_0 e^{\beta p}$  for the entrance zone give

$$M' + \varphi(x) M = \psi(x); \quad M = \frac{1}{\eta} \frac{\partial p}{\partial x}, \quad (3)$$

where

$$\varphi(x) = \gamma + \frac{1}{h^3} \frac{d(h^3)}{dx}; \quad \psi(x) = -6 \frac{dh}{dx} \left( \frac{V_1 + \bar{V}_t}{h^3} \right).$$

The solution to (3) takes the form [7]

$$M = C_1 \exp \left[ - \int \varphi(x) dx \right] + \exp \left[ - \int \varphi(x) dx \right] \int \psi(x) \exp \left[ \int \varphi(x) dx \right] dx, \quad (5)$$

where  $C_1$  is a constant of integration; we substitute (4) into (5) and integrate subject to the boundary conditions

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$$h = h_2, \quad p = p_1, \quad h = h_1, \quad p = p_1 + \sigma_3$$

to get an equation for the thickness of the lubricant layer at the end of the entrance zone [2]:

$$h_1^2 - (Bh_2^2 + 2h_2)h_1 + h_2^2 = 0, \quad (6)$$

where

$$B = \frac{\exp(-\beta p_1) [1 + \exp(-\beta \sigma_3)] \operatorname{tg} \alpha}{3\eta_0 \beta [V_1 + \bar{V}_t]},$$

where  $\eta_0$  and  $\beta$  are the viscosity and pressure-response coefficient of the liquid at atmospheric pressure and the environmental temperature,  $p_1$  is the pressure required in the liquid during extrusion, and  $\sigma_3$  is the yield point in the presence of work-hardening:

$$\sigma_3 = \sigma_0 + a\varepsilon_{av}^n, \quad (7)$$

where  $\sigma_0$  is the yield point of the annealed material,  $a$  and  $n$  are the work-hardening coefficients, and  $\varepsilon_{av}$  is the average strain.

The distributions of the contact stresses and pressure in the plastic-strain zone (zone 2) may be derived by considering an infinitely small component of the metal of thickness  $dx$  (Fig. 2). The equation of equilibrium for this in the (XOY) coordinate system takes the following form [1] when the plasticity condition is incorporated:

$$\frac{X}{2} \frac{\partial p}{\partial X} - \sigma_s - \frac{\tau}{\operatorname{tg} \alpha} = 0, \quad (8)$$

where  $\tau$  is the tangential stress at the surface of the metal in the deformation focus, which is given [8] by

$$\tau = -\eta \left( \frac{\partial u}{\partial Y} \right)_{Y=h^*}. \quad (9)$$

The equation of equilibrium for the liquid in this zone is

$$\frac{\partial p}{\partial X} = \eta \frac{\partial^2 u}{\partial Y^2}. \quad (10)$$

The thickness of the lubricant layer is negligible, so we can assume that the viscosity is independent of  $Y$  in determining the speed of the lubricant.

Double integration of (10) with the boundary conditions

$$Y=0, \quad u = -\bar{V}_t, \quad Y=h^*, \quad u = -V_1 \left( \frac{X_1}{X} \right)^2$$

gives

$$u = \frac{1}{2\eta} \frac{\partial p}{\partial X} (Y^2 - h^*Y) - \frac{V_1 Y}{h^*} \left( \frac{X_1}{X} \right)^2 - \bar{V}_t \cos \alpha \left( 1 - \frac{Y}{h^*} \right). \quad (11)$$

Substitution of (11) and (8) with  $h^* = h_1^* X/X_1$  gives

$$\tau = -\frac{h_1^*}{2} \left( \frac{X_1}{X} \right) \frac{\partial p}{\partial X} + \frac{V_1}{h_1^*} \eta \left( \frac{X_1}{X} \right)^3 - \frac{\eta \bar{V}_t \cos \alpha}{h_1^*} \left( \frac{X_1}{X} \right), \quad (12)$$

where  $h_1^*$  is the thickness of the lubricant layer at the exit from the deformation zone (Fig. 2).

We assume that the viscosity of the lubricant in the plastic-strain zone varies as follows with pressure and temperature [6]:

$$\eta = \eta_0 \exp [\beta p - b(t_i - t_0)]. \quad (13)$$

To determine  $t_i$  we use the equation for quasistationary heat transfer in the lubricant layer [9]:

$$\lambda_e \frac{\partial^2 t}{\partial Y^2} = \eta \left( \frac{\partial u}{\partial Y} \right)^2. \quad (14)$$

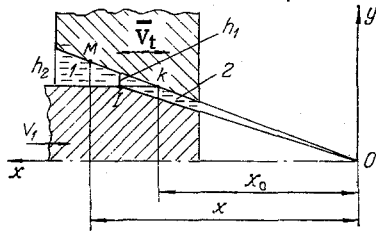


Fig. 1

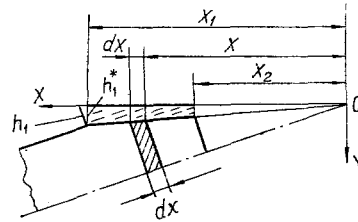


Fig. 2

Fig. 1. Determination of the lubricant layer thickness at the entry to the strain focus.

Fig. 2. State of strain during hydraulic extrusion of a metal.

Substitution of (11) into (14) and solution for  $t$  with the boundary conditions  $Y = 0$ ,  $t = t_a$  and  $Y = h^*$ ,  $t = t_b$  give

$$t = -\frac{1}{4\eta\lambda_e} \left( \frac{\partial p}{\partial X} \right)^2 \left[ \frac{Y^4}{3} - \frac{2h^*Y^3}{3} + \frac{h^{*2}Y^2}{4} - \frac{h^{*3}Y}{6} \right] - \frac{\eta}{\lambda_e} \left[ \frac{V_1}{h^*} \left( \frac{X_1}{X} \right)^2 - \bar{V}_t \right]^2 \frac{Y}{2} (Y - h^*) +$$

$$+ \frac{1}{\lambda_e} \frac{\partial p}{\partial X} Y \left[ \frac{V_1}{h^*} \left( \frac{X_1}{X} \right)^2 - \bar{V}_t \right] \left[ \frac{Y^2}{3} - \frac{h^*Y}{2} + \frac{h^{*2}}{6} \right] + \frac{Y}{h^*} t_b + t_a \left( 1 - \frac{Y}{h^*} \right). \quad (15)$$

As the lubricant film is thin, it is sufficient to determine the mean temperature

$$t_i = \frac{1}{h^*} \int_0^{h^*} t dy = \frac{h^{*4}}{240\eta\lambda_e} \left( \frac{\partial p}{\partial X} \right)^2 + \frac{\eta}{12\lambda_e} \left[ V_1 \left( \frac{X_1}{X} \right)^2 - \bar{V}_t \right]^2 + \frac{1}{2} (t_a + t_b). \quad (16)$$

The metal is cooled only slightly by heat transfer, so the above assumptions imply that the temperature of the metal at the deformation focus is defined by

$$t_b = t_0 + \frac{A}{c\gamma} + \frac{BT_1}{\gamma V_0 c T}. \quad (17)$$

The work done  $A$  is [9] given by

$$A = \int_0^{\varepsilon_{av}} \sigma_s d\varepsilon_{av} = \sigma_0 \varepsilon_{av} + a \frac{\varepsilon_{av}^{(n+1)}}{(n+1)}. \quad (18)$$

The average strain is given by

$$\varepsilon_{av} = \frac{4}{3\sqrt{3}} \operatorname{tg} \alpha + \frac{16 \operatorname{Ln}(D_0/D_1)}{9 \operatorname{tg}^2 \alpha} \left[ \left( 1 + \frac{3}{4} \operatorname{tg}^2 \alpha \right)^{3/2} - 1 \right].$$

The mean ultrasonic energy is

$$\bar{B} = \frac{1}{2} \rho \bar{V}_t^2 \sin^2 \alpha V_{\text{wave}} D, \quad (19)$$

where  $D$  is the energy-transmission factor, which itself is [10] defined by

$$D = \frac{W_1 W_2}{(W_1 + W_2)^2} = \frac{\rho c_1 \gamma c_2}{(\rho c_1 + \gamma c_2)^2}. \quad (20)$$

The time is

$$T_1 = (X_1 - X_2) \bar{V}. \quad (21)$$

The mean speed of the metal in the deformation focus is

$$\bar{V} = \frac{1}{X_1 - X_2} \int_{X_2}^{X_1} V_1 \left( \frac{X_1}{X} \right)^2 dX = V_1 \frac{X_1}{X_2} \quad (22)$$

Then (22) gives (21) the form

$$T_1 = X_2(X_1 - X_2)/V_1 X_1. \quad (23)$$

Substitution of (18) and (19) into (17) gives

$$t_b = t_0 + \frac{\sigma_0 \varepsilon_{av} + a\varepsilon_{av}^{(n+1)}/(n+1)}{c\gamma} + \frac{1}{2} \frac{\rho \bar{V}_t^2 \sin^2 \alpha V_{wm} D T_1}{\gamma V_0 c T} \quad (24)$$

The temperature  $t_a$  of the die is determined by experiment.

Then the temperature  $t_i$  of the lubricant layer is given by

$$t_i = \frac{h^{*4}}{240\eta\lambda_e} \left( \frac{\partial p}{\partial X} \right)^2 + \frac{\eta}{12\lambda_e} \left[ V_1 \left( \frac{X_1}{X} \right)^2 - \bar{V}_t \right]^2 + \frac{1}{2} (t_0 + t_a) + \frac{1}{2} \left[ \frac{\sigma_0 \varepsilon_{av} + a\varepsilon_{av}^{(n+1)}/(n+1)}{c\gamma} + \frac{1}{2} \frac{\rho \bar{V}_t^2 \sin^2 \alpha V_{wm} D T_1}{\gamma V_0 c T} \right] \quad (25)$$

We neglect the first term in (25) because it is small by comparison with the others, which gives

$$t_i = \frac{\eta}{12\lambda_e} \left[ V_1 \left( \frac{X_1}{X} \right)^2 - \bar{V}_t \right]^2 + \frac{1}{2} (t_0 + t_a) + \frac{1}{2} \left[ \frac{\sigma_0 \varepsilon_{av} + a\varepsilon_{av}^{(n+1)}/(n+1)}{c\gamma} + \frac{1}{2} \frac{\rho \bar{V}_t^2 \sin^2 \alpha V_{wm} D T_1}{\gamma V_0 c T} \right] \quad (26)$$

Substitution of (26) into (13) gives

$$\eta - \eta_0 \exp \left\{ \left\{ \beta p - b \left[ \frac{\eta}{12\lambda_e} \left[ V_1 \left( \frac{X_1}{X} \right)^2 - \bar{V}_t \right]^2 + \frac{1}{2} (t_a - t_0) + \frac{1}{2} \left[ \frac{\sigma_0 \varepsilon_{av} + a\varepsilon_{av}^{(n+1)}/(n+1)}{c\gamma} + \frac{1}{2} \frac{\rho \bar{V}_t^2 \sin^2 \alpha V_{wm} D T_1}{\gamma V_0 c T} \right] \right] \right\} \right\} \quad (27)$$

We solve (27) subject to the conditions  $X = X_1$ ,  $p = \sigma_s + p_1$  and  $X = X_2$ ,  $p = \sigma_s$  to get the values for the dynamic viscosity  $\eta_1$  and  $\eta_2$  at the entrance and exit to the plastic-strain zone, and these are substituted into (26) to get the corresponding temperatures of the lubricant layer:

$$t_{ij} = \frac{\eta_j}{12\lambda_e} \left[ V_1 \left( \frac{X_1}{X_j} \right)^2 - \bar{V}_t \right]^2 + Q, \quad (28)$$

where  $j = 1, 2$ ;

$$Q = \frac{1}{2} (t_a - t_0) + \frac{1}{2} \left[ \frac{\sigma_0 \varepsilon_{av} + a\varepsilon_{av}^{(n+1)}/(n+1)}{c\gamma} + \frac{1}{2} \frac{\rho \bar{V}_t^2 \sin^2 \alpha V_{wm} D T_1}{\gamma V_0 c T} \right].$$

The plastic-strain zone is short when a metal is extruded in the presence of ultrasound, so the mean temperature at any point in the lubricant layer within the strain focus is given to a first approximation by

$$\bar{t}_i = \frac{t_{i1} + t_{i2}}{2} = \frac{1}{24\lambda_e} \left\{ \eta_1 (V_1 - \bar{V}_t)^2 + \eta_2 \left[ V_1 \left( \frac{X_1}{X_2} \right)^2 - \bar{V}_t \right]^2 \right\} + Q. \quad (29)$$

We substitute  $\bar{t}_i$  for  $t_i$  in (12) to get

$$\eta = \eta_0 \exp [-b(\bar{t}_i - t_0)] \exp(\beta p) = \eta_3 \exp(\beta p), \quad (30)$$

$$\eta_3 = \eta_0 \exp [-b(\bar{t}_i - t_0)].$$

Substitution of (30) and (12) into (8) gives

$$\frac{X}{2} \left( 1 + \frac{h_1^*}{X_1 \tan \alpha} \right) \frac{\partial p}{\partial X} - \sigma_s - \left\{ \frac{\eta_3 V_1}{h_1^*} \left( \frac{X_1}{X} \right)^3 - \frac{\eta_3 \bar{V}_t \cos \alpha}{h_1^*} \left( \frac{X_1}{X} \right) \right\} \frac{\exp(\beta p)}{\tan \alpha} = 0. \quad (31)$$

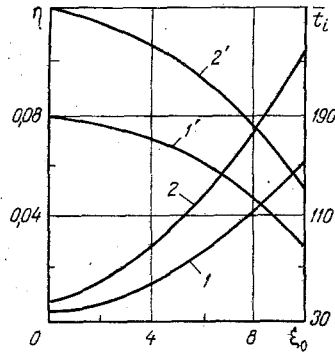


Fig. 3

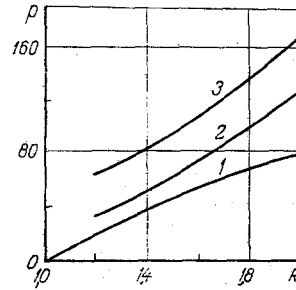


Fig. 4

Fig. 3. Temperature and dynamic viscosity of lubricant at the deformation focus in relation to oscillation amplitude during the hydraulic extrusion of aluminum in the presence of spindle oil; 1, 2) temperature; 1', 2') viscosity; 1, 1')  $R = 1.21$ ; 2, 2')  $R = 1.69 \cdot \eta$ ,  $N \cdot \text{sec}/\text{m}^2$ ;  $\xi_0 \cdot \mu\text{m}$ ;  $\bar{t}_i$ ,  $^{\circ}\text{C}$ .

Fig. 4. Pressure in liquid in relation to drawing factor in hydraulic extrusion of aluminum in the presence of longitudinal ultrasound: 1) theoretical result for ultrasound; 2) experimental result for ultrasound; 3) experimental result without ultrasound;  $p$  in  $\text{MN}/\text{m}^2$ .

As  $h_1^*/X_1 \tan \alpha \ll 1$ , we neglect this quantity, and then (31) becomes

$$\frac{X}{2} \frac{\partial p}{\partial X} - \sigma_s - \left\{ \frac{\eta_3 V_1}{h_1^*} \left( \frac{X_1}{X} \right)^3 - \frac{\eta_3 \bar{V}_t \cos \alpha}{h_1^*} \left( \frac{X_1}{X} \right) \right\} \frac{\exp(\beta p)}{\text{tg } \alpha} = 0. \quad (32)$$

We solve (32) with the boundary conditions

$$X = X_1, \quad p = p_1 + \sigma_s \text{ and } X = X_2, \quad p = \sigma_s$$

to get

$$\exp(-\beta \sigma_s) = q \left[ \left( \frac{X_1}{X_2} \right)^3 - \left( \frac{X_1}{X_2} \right)^{2G} \right] + s \left[ \frac{X_1}{X_2} - \left( \frac{X_1}{X_2} \right)^{2G} \right] + \left( \frac{X_1}{X_2} \right)^{2G} \exp[-\beta(p_1 + \sigma_s)]. \quad (33)$$

Here

$$G = \beta \sigma_s; \quad q = -\frac{2\eta_3 \beta V_1}{h_1^* \text{tg } \alpha (2G - 3)}; \quad s = \frac{2\eta_3 \beta \bar{V}_t \cos \alpha}{h_1^* \text{tg } \alpha (2G - 1)}; \quad h_1^* = h_1 \cos \alpha.$$

We divide both sides in (33) by  $\exp[-\beta(p_1 + \sigma_s)]$  and transform the result to get

$$p_1 - \sigma_s \ln R - \frac{1}{\beta} \ln \left\{ 1 + q \left( R^{\frac{3-2G}{2}} - 1 \right) \exp[\beta(p_1 + \sigma_s)] + s \left( R^{\frac{1-2G}{2}} - 1 \right) \exp[\beta(p_1 + \sigma_s)] \right\} = 0. \quad (34)$$

Then (34) gives the working pressure  $p_1$  in the liquid for a metal extruded in the presence of longitudinal ultrasound.

Three separate steps have to be considered in order to determine  $p_1$ : a) the thickness  $h_1$  of the lubricant in the entrance zone for a given pressure  $p_1 = p_{11}$  in the liquid; b) use of this result to find  $\bar{t}_1$  and  $\eta_3$ ; and c) substitution of  $\bar{t}_1$ ,  $\eta_3$ ,  $h_1^*$  ( $h_1^* = h_1 \cos \alpha$ ) into (34) to find  $p_1$ . The result for  $p_1$  is compared with the specified  $p_{11}$ ; if there is any difference, a new value  $p_1 = p_{11} + \Delta p_{11}$  is assumed for the pressure. The calculations on  $h_1$ ,  $\bar{t}_1$ ,  $\eta_3$ ,  $p_1$  are repeated until the result agrees with the specified value. This method of determining  $p_1$ ,  $h_1$ ,  $\eta_3$ ,  $\bar{t}_1$  has been used in a Minsk-32 program in which the input data include specifications for the metal and other parameters ( $R$ ,  $V_1$ ,  $\bar{V}_t$ ,  $t_0$ ,  $t_a$ ,  $\eta_0$ ,  $\gamma$ ,  $b$ , ...).

During the second half of the period, the liquid begins to be compressed, while the thickness of the lubricant layer falls to its minimum value. Then the pressure in the liquid is given [11] by

$$p_{1\text{max}} = \frac{\left[ \sigma_s (d + 1) - \left( 1 + \frac{1}{\sin \alpha} \right) \sigma_v \right]}{d} [R - 1], \quad (35)$$

where  $d = \mu/\tan\alpha$ ; and  $\sigma_v$  is the oscillatory stress in the waveguide at the point  $X_{av} = (X_1 + X_2)/2$ .

The mean pressure in the liquid during the extrusion is given

$$p_{lav} = \frac{p_1 + p_{1max}}{2}.$$

Figures 3 and 4 give the theoretical results along with measurements.

These results show that the working pressure is reduced by ultrasound during hydraulic extrusion; the surface finish is improved by two grades.

#### NOTATION

$h, h^*$	are the lubricant thicknesses at points in the inlet zone and in deformation region, respectively;
$h_1, h_2$	are the same at inlet and outlet of the first zone, respectively;
$(x_0y), (X_0Y)$	are the coordinate systems for lubricant thickness and pressure in the deformation region;
$x, x_0$	are the coordinates of points $m$ and $k$ ;
$\alpha$	is the semivertex angle of die;
$V_1, \bar{V}, V_t$	are the speed of metal, mean speed in deformation region, and mean speed of tool;
$p, p_1$	are the pressure in the deformation region and in the source, respectively;
$\eta_0, \eta$	are the dynamic viscosities in free and working states, respectively;
$\tau$	is the shear stress;
$M = (1/\eta)(\partial p/\partial x)$ ; $u$	is the velocity of lubricating film;
$\varepsilon_{av}$	is the mean strain;
$a, n$	are the work-hardening factors;
$\sigma_s, \sigma_0$	are the yield points with and without hardening;
$\beta, b$	are the pressure and temperature coefficients of viscosity;
$t_a, t_b, t$	are the temperatures of the tool, of the metal in the deformation region, and of the lubricant, respectively;
$t_0, t_1, \bar{t}_i$	are the temperature in the free state, mean temperature of lubricant at a point in the deformation region, and mean temperature over the deformation region;
$\lambda_e$	is the thermal conductivity of lubricant;
$c, \gamma$	are the specific heat and density;
$A$	is the work of plastic strain;
$\bar{B}$	is the ultrasonic energy entering deformation region;
$T, T_1$	are the period of oscillation and time of motion from entrance to exit of deformation region;
$V_0, V_{wm}$	are the volumes of metal in the deformation region and in the surrounding guide;
$\rho$	is the density of guide material;
$c_1, c_2$	are the speeds of longitudinal waves in guide and metal, respectively;
$W_1, W_2$	are the specific acoustic impedances of guide and metal;
$D$	is the coefficient of the energy transfer from the waveguide to the material;
$D_0, D_1$	are the diameters of workpiece and product;
$X_1, X_2$	are the coordinates of the entrance and exit to the deformation region in the $(X_0Y)$ system;
$R$	is the deformation factor;
$\sigma_v$	is the oscillatory stress;
$\mu$	is the coefficient of viscous friction;
$p_1 \max$	is the pressure in conventional extrusion;
$p_1 \text{ av}$	is the mean pressure in hydraulic extrusion.

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## EFFECT OF DEFECTIVE FUEL ELEMENT PARAMETERS ON TEMPERATURE DISTRIBUTIONS

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Results are presented of the calculation of temperature distributions in the cross section of fuel rods with a defective contact bond. The effects of the dimensions and thermal conductivity of the defects, the dimensions of the fuel element, the heat-conduction properties of the fuel element core and cladding, and the rate of cooling are determined.

The effects of the thermal conductivity and the dimensions of defects, and the dimensions, heat-conduction properties, and rate of cooling of defective fuel rods on temperature distributions are established.

During the manufacture and use of fuel elements various defects may develop, the most important of which are local impairments of heat transfer between the core and cladding. Such defects lead to a distortion of the temperature distribution, which in turn gives rise to thermal stresses, acceleration of corrosion processes, and a loss of mechanical strength. In view of this it is necessary to develop methods for calculating temperature distributions in defective fuel elements to determine the controlling factors which have an appreciable effect on the temperature distribution.

The most accurate determination of the temperature distribution would involve the simultaneous solution of the differential equations describing the temperature distribution in the fuel element and in the coolant. However, because of the difficulty of describing the velocity distribution and the turbulent component of heat conduction most calculations have been performed by solving the heat-conduction equations in the fuel element with boundary conditions of the third kind specified at the boundary between the fuel element and the coolant.

Calculational methods or the results of temperature calculations for certain special problems have been published [1-4], but the basic rules for the effect of dimensions and thermal parameters of fuel elements and defects on the distribution of temperature fields have not been established.

If these rules were known the construction of fuel elements could be optimized so as to minimize the effect of their defects on their operating life.

With this in mind we have performed numerical calculations of temperature distributions in the cross section of a fuel rod with an infinitely long defect in the contact bond between the core and cladding.

For constant physical parameters and dimensions of the fuel element and defect, and a constant coefficient of heat transfer from the surface of the fuel element the dimensionless equations describing the steady temperature distribution in the cross section of a fuel element with a defect of width  $2\varphi R_1$  have the form

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